Math 111I
Joseph C Foster
Fall 2017
Final Exam

Name: _____

December 12th, 2017
Time Limit: 150 minutes

This exam contains 10 pages (including this cover page) and 30 questions.
The total number of marks is 100. You have 150 minutes to complete the exam.

Read each question carefully. When specified, you must show **all** *necessary* work to receive full credit.

No calculator/phone/smartwatch allowed under any circumstances. Place these items in your bag, out of reach. Cheating of any kind will not be tolerated and will result in a grade of zero.

Question	Marks	Score	Question	Marks	Score
1	2		16	4	
2	2		17	4	
3	2		18	4	
4	2		19	4	
5	2		20	4	
6	2		21	4	
7	2		22	4	
8	2		23	4	
9	2		24	4	
10	2		25	5	
11	2		26	5	
12	2		27	5	
13	2		28	6	
14	2		29	6	
15	2		30	7	
			Total	100	

1. (2 marks) True or False: $2xy - 3y = x(2y - 3y)$

     A. True   B. False

2. (2 marks) True or False: $x^2 - 4x - 21 = (x + 7)(x - 3)$.

     A. True   B. False

3. (2 marks) True or False: $(A - B)(A + B) = A^2 - B^2$

     A. True   B. False

4. (2 marks) True or False: If $x = y$ then $f(x) = f(y)$.

     A. True   B. False

5. (2 marks) True or False: For two functions $f(x)$ and $g(x)$, if $f(g(x)) = x$ then $f(x)$ and $g(x)$ are inverses of each other.

     A. True   B. False

6. (2 marks) Fill in the blank: Given a quadratic equation $ax^2 + bx + c = 0$, the roots are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

7. (2 marks) Fill in the blank: The quadratic equation $5x^2 - 17x - 600 = 0$ has __2__ solutions.

8. (2 marks) Fill in the blank: The *domain* of a function $f(x)$ is the set of all input values for $f(x)$

_____.

9. (2 marks) Fill in the blank: The *range* of a function $f(x)$ is the set of all output values for $f(x)$

_____.

10. (2 marks) Fill in the blank: The lines $y_1 = mx + b$ and $y_2 = nx + c$ are *parallel* if

$$m = n$$

11. (2 marks) Fill in the blank: The lines $y_1 = mx + b$ and $y_2 = nx + c$ are *perpendicular* if

$$mn = -1$$

For questions 12-15, choose the best answer. There is only one correct answer but you may choose up to *two*. If you choose two and one of the answers is correct, you will receive 1 mark.

12. (2 marks) An example of an irrational number is:

   A. 0

   B. $\sqrt{4}$

   C. $-\dfrac{3}{2}$

   D. $\sqrt{2}$

13. (2 marks) In an exponential model $P(t) = Ca^t$, what is the relationship between a and r?

   A. $a = r - 1$

   B. $r = 1 - a$

   C. $r = a - 1$

   D. $a = r$

14. (2 marks) If an initial amount of P dollars is invested in an account with an annual interest rate of r, compounded n times a year, which of the following represents the amount in the account after t years.

   A. $A(t) = P(1 + r)^{nt}$

   B. $A(t) = P\left(1 + \dfrac{r}{n}\right)^{t}$

   C. $A(t) = P\left(1 + \dfrac{r}{n}\right)^{nt}$

   D. $A(t) = Pe^{rt}$

15. (2 marks) If $f(x) = x^2 + 5$, then

   A. $f^{-1}(x) = \sqrt{x - 5}$

   B. $f^{-1}(x) = \sqrt{x} - 5$

   C. $f^{-1}(x) = \sqrt{x} - \sqrt{5}$

   D. None of these.

16. (4 marks) Fill in the table below. You may assume that all letters represent positive numbers.

$a^m a^n = a^{m+n}$	$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$	$a^{1/n} = \sqrt[n]{a}$
$\dfrac{a^m}{a^n} = a^{m-n}$	$a^0 = 1$	$a^{m/n} = \sqrt[n]{a^m}$
$(a^m)^n = a^{mn}$	$a^{-n} = \dfrac{1}{a^n}$	$(ab)^n = a^n b^n$

17. (4 marks) If a, r, s, t are all positive real numbers, then;

$f(x-t)$ is a <u>shift</u> of $f(x)$ <u>t</u> units to the <u>right</u>.
$af(x)$ is a <u>stretch</u> of $f(x)$ in the <u>y-direction</u>, by a scale factor of <u>a</u>.
$f(x) + s$ is a <u>shift</u> of $f(x)$ <u>s</u> units <u>up</u>.
$f(-x)$ is a <u>reflection</u> of $f(x)$ in the <u>y-axis</u>.

For questions 18-30, show **all** *necessary* work to receive full credit. Please circle or box your final answer. If you cannot complete a problem but can write down what you want to do, and this is correct, you can still receive partial credit. Don't leave anything blank!

18. (4 marks) Evaluate the arithmetic expression

$$\frac{5+7}{3} - 6\left[12 - (17 - 2 \cdot 3)\right]$$

$$= \frac{12}{3} - 6\left[12 - (17 - 6)\right]$$
$$= 4 - 6\left[12 - 11\right]$$
$$= 4 - 6\left[1\right]$$
$$= 4 - 6$$
$$= \boxed{-2}$$

19. (4 marks) Rationalise the denominator of the fraction

$$\frac{2(x-y)}{\sqrt{x} - \sqrt{y}}$$

and simplify your answer fully.

$$= \frac{2(x-y)\left(\sqrt{x} + \sqrt{y}\right)}{\left(\sqrt{x} - \sqrt{y}\right)\left(\sqrt{x} + \sqrt{y}\right)}$$
$$= \frac{2(x-y)\left(\sqrt{x} + \sqrt{y}\right)}{x - y}$$
$$= \boxed{2\left(\sqrt{x} + \sqrt{y}\right)}$$

20. (4 marks) Perform the indicated operation and simplify your answer fully.

$$\left(3ab\sqrt[5]{16a^4b^2}\right) \cdot \left(8b^7 \sqrt[5]{2ab^3}\right).$$

$$= 24ab^8 \sqrt[5]{16a^4b^2 2ab^3}$$
$$= 24ab^8 \sqrt[5]{32a^5b^5}$$
$$= 24ab^8 2ab$$
$$= \boxed{48a^2b^9}$$

21. (4 marks) What is the slope of the linear function $f(x)$ passing through the points $(3,5)$ and $(6,9)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 5}{6 - 3} = \boxed{\frac{4}{3}}$$

22. (4 marks) Give the equation of the line with slope 5 that goes through the point $(-2, 3)$. You may leave your answer in either point-slope or slope-intercept form.

$$\boxed{y - 3 = 5\,(x + 2)}$$

23. (4 marks) Let $f(x) = \sqrt{x}$. Find the expression of the function $g(x)$ whose graph is obtained by the following transformations on $f(x)$.

   1. Shift left 2 units.
   2. Reflect about the $x-$axis.
   3. Shift up 6 units.

   1. $\sqrt{x + 2}$
   2. $-\sqrt{x + 2}$
   3. $\boxed{-\sqrt{x + 2} + 6}$

24. (4 marks) Find the domain of the function

$$f(x) = \frac{\sqrt{x + 2}}{x - 5}.$$

Numerator: $x \geq -2$

Denominator: $x \neq 5$

Domain: $\boxed{[-2, 5) \cup (5, \infty)}$
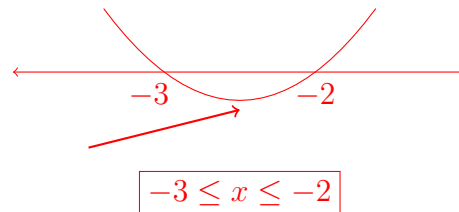
25. (5 marks) Solve the following inequality

$$\log_2\left(x^2 + 5x + 14\right) \leq 3.$$

$\Rightarrow \log_2\left(x^2 + 5x + 14\right) \leq \log_2(8)$

$\Rightarrow x^2 + 5x + 14 \leq 8$

$\Rightarrow x^2 + 5x + 6 \leq 0$

$\Rightarrow (x+2)(x+3) \leq 0$

$$\boxed{-3 \leq x \leq -2}$$

26. (5 marks) Find *all* roots of the following polynomial equation. *(Hint: simplify first.)*

$$(x-1)(x^2 - 6x - 4) + (x-1)5x = 0.$$

$\Rightarrow (x-1)\left(x^2 - 6x - 4 + 5x\right) = 0$

$\Rightarrow (x-1)\left(x^2 - x - 4\right) = 0$

$\Rightarrow x - 1 = 0$ or $x^2 - x - 4 = 0$

$\Rightarrow x = 1$ or $x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-4)}}{2(1)}$

$\Rightarrow x = 1$ or $\dfrac{1 \pm \sqrt{1 + 16}}{2}$

$\Rightarrow \boxed{x = 1 \text{ or } \dfrac{1 \pm \sqrt{17}}{2}}$

27. Internet World Stats reports that the number of internet users in China increased by 1500% from 2013 to 2017 (so the 4 year growth rate is 15 and the 4 year growth factor is 16). The number of internet users in 2013 was 17.5 million. Assume that the number of internet users increases exponentially.

  (a) (2 marks) Find the *yearly* growth factor a and find an exponential model $f(t) = Ca^t$ for the number of internet users in China after t years since 2013, where the number of users is measured in millions.

$$1 \text{ year} = \frac{1}{4} \times 4\text{years}$$
$$a = 16^{1/4} = 2$$

$$\boxed{f(t) = 17.5\left(2^t\right)}$$

  (b) (3 marks) In what year would you expect the number of internet users in China to reach 560 million?

$$17.5\left(2\right)^t = 560$$
$$\Rightarrow 2^t = 32$$
$$\Rightarrow t = 5$$
$$\Rightarrow \boxed{2018}$$

28. (6 marks) Determine the equation of the quadratic function whose graph is given below. Each square represents 1 unit.

$$f(x) = a\left(x+3\right)^2 - 4$$
$$f(-4) = a\left(-4+3\right)^2 - 4$$
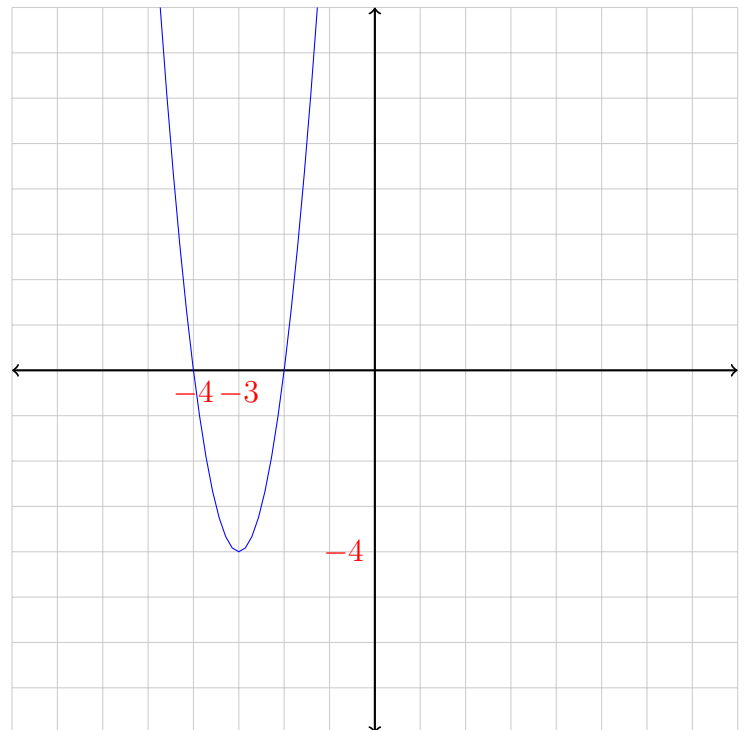$$= a\left(-1\right)^2 - 4$$
$$= a - 4$$
$$= 0$$
$$\Rightarrow a = 4$$

$$\Rightarrow \boxed{f(x) = 4\left(x+3\right)^2 - 4}$$

29. Henry wants to buy a new car, and he has narrowed his choices to two models.

Model A sells for $\$12,000$, gets 25mi/gal, and costs $\$300$ a year for insurance
Model B sells for $\$15,000$, gets 40mi/gal, and costs $\$425$ a year for insurance

Henry drives about $25,000$ miles per year, and gas costs about $\$3$ a gallon.

(a) (2 marks) Find a function $A(t)$ that models the total cost $A(t)$ of owning Model A for t years.

$$A(t) = \text{Initial Cost} + (\text{Cost per Year})\, t$$
$$= \text{Initial Cost} + \left(\text{Insurance} + \frac{\text{Miles Driven}}{\text{Miles per Gallon}} \times \text{Cost per Gallon}\right) t$$
$$= 12,000 + \left(300 + \frac{25,000}{25}3\right) t$$
$$= 12,000 + (300 + 3,000)\, t$$

$$\Rightarrow \boxed{A(t) = 12,000 + 3,300t}$$

(b) (2 marks) Find a function $B(t)$ that models the total cost $B(t)$ of owning Model B for t years.

$$B(t) = \text{Initial Cost} + \left(\text{Insurance} + \frac{\text{Miles Driven}}{\text{Miles per Gallon}} \times \text{Cost per Gallon}\right) t$$
$$= 15,000 + \left(425 + \frac{25,000}{40}3\right) t$$
$$= 15,000 + (425 + 1875)\, t$$

$$\Rightarrow \boxed{B(t) = 15,000 + 2,300t}$$

(c) (2 marks) Find the number of years of ownership for which the cost to Henry of owning Model A equals the cost of owning Model B.

$$A(t) = B(t)$$
$$12,000 + 3,300t = 15,000 + 2,300t$$
$$1,000t = 3,000$$
$$\boxed{t = 3 \text{ years}}$$

30. (a) (4 marks) Simplify the following expression completely,

$$\frac{3x^2 + 6x - 105}{x^2 - 2x - 15}.$$

$$= \frac{3\left(x^2 + 2x - 35\right)}{(x - 5)(x + 3)}$$

$$= \frac{3(x - 5)(x + 7)}{(x - 5)(x + 3)}$$

$$= \boxed{\frac{3(x + 7)}{x + 3}}$$

(b) (3 marks) Use polynomial long division to find the quotient and remainder of

$$\frac{(x^2 + x - 6)(3x^2 + 6x - 105)(x^2 - 11x + 13)}{3(x - 2)(x + 7)(x^2 - 2x - 15)(x - 11)}.$$

Use part (a) to help you.

$$= \frac{(x^2 + x - 6)\,3(x + 7)(x^2 - 11x + 13)}{3(x - 2)(x + 7)(x + 3)(x - 11)}$$

$$= \frac{(x + 3)(x - 2)\,3(x + 7)(x^2 - 11x + 13)}{3(x - 2)(x + 7)(x + 3)(x - 11)}$$

$$= \frac{x^2 - 11x + 13}{x - 11}$$

$$
\begin{array}{r}
x \\
x - 11 \overline{)\ x^2 - 11x + 13} \\
\underline{-\ x^2 + 11x} \\
13
\end{array}
$$

$$\boxed{x + \frac{13}{x - 11}}$$